

UNIT-5

MEASUREMENT OF FLOW

* Venturimeter :

A Venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

- (i) A Short Converging part
- (ii) Throat
- (iii) Diverging part

It is based on the principle of Bernoulli's equation.

Expression for rate of flow through venturimeter

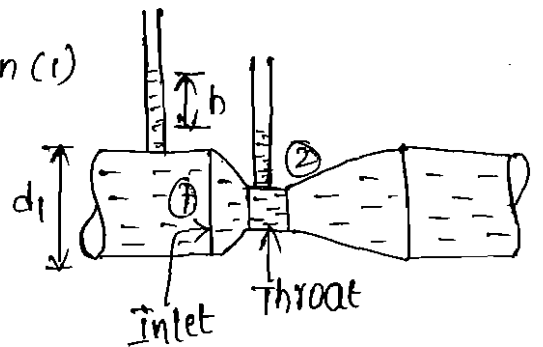
Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water) as shown in fig.

Let d_1 = diameter at inlet or at section (1)

P_1 = pressure at section (1)

v_1 = Velocity of fluid at section (1)

a = Area at section (1) = $\frac{\pi}{4} d_1^2$



and d_2, P_2, v_2, a_2 are corresponding values at section (2), Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h

$$\frac{P_1 - P_2}{\rho g} = h$$

Substitute in above equation

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{--- (1)}$$

Now applying Continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

Substitute in (1)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g}$$

$$= \frac{v_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$= \frac{v_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh \left(\frac{a_1^2}{a_1^2 - a_2^2}\right)} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\text{Discharge } Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q_{\text{actual}} = C_d \times Q_{\text{th}} \quad (2)$$

$$= C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where C_d = Coefficient of venturimeter and its value is less than 1.

Note:

1. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = sp. gravity of heavier liquid

S_o = sp. gravity of liquid flowing through pipe

x = difference of heavier liquid column in U-tube

$$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

2. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x \left[1 - \frac{S_l}{S_o} \right]$$

where S_l = sp. gravity of lighter liquid

S_o = sp. gravity of liquid flowing through pipe.

x = difference of lighter liquid column in U-tube

3. Inclined Venturimeter with differential U-tube manometer:

The above two cases are given for a horizontal venturimeter. This

case is related to inclined venturimeter having differential U-tube manometer.

Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = x \left[\frac{S_h}{S_o} - 1 \right]$$

4) Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = x \left[1 - \frac{S_l}{S_o} \right]$$

problems

1. A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.

Sol:-

Given :

Dia. at inlet, $d_1 = 30 \text{ cm}$

\therefore Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15 \text{ cm}$

$a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = $x = 20 \text{ cm}$ of mercury

\therefore Difference of pressure head is $h = x \left[\frac{S_h}{S_o} - 1 \right]$

$$S_h = \text{sp. gravity of mercury} = 13.6 \quad (3)$$

$$S_o = \text{sp. gravity of water} = 1$$

$$h = 20 \left[\frac{13.6}{1} - 1 \right] = 20 \times 12.6 \text{ cm} = 252 \text{ cm of water.}$$

The discharge through venturimeter is

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}}$$

$$= 125756 \text{ cm}^3/\text{s}$$

$$= \frac{125756}{1000} \text{ lit/s}$$

$$\boxed{Q = 125.756 \text{ lit/s}}$$

2. A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm^2 and vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter.

Take $C_d = 0.98$.

Sol:-

Given

dia. at inlet $d_1 = 20 \text{ cm}$

$$a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$$

dia. at throat $d_2 = 10 \text{ cm}$

$$a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$$

$$P_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$$

$$\frac{P_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18 \text{ m of water}$$

$$\frac{P_2}{\rho g} = -30 \text{ cm of mercury}$$

$$= -0.30 \text{ m of mercury}$$

$$= -0.3 \times 13.6$$

$$= -4.08 \text{ m of water}$$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 18 - (-4.08) = 22.08 \text{ m of water}$$

$$= 2208 \text{ cm of water}$$

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= 165555 \text{ cm}^3/\text{s}$$

$$Q = 165.555 \text{ lit/s}$$

Orifice meter or orifice plate:

It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe.

The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter. (4)

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2 times the pipe diameter upstream from the orifice plate and at section (2), which is at a distance of about half the diameter of orifice on downstream side from the orifice plate.

Let p_1 = pressure at section (1)

v_1 = velocity at section (1)

a_1 = area of pipe at section (1)

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

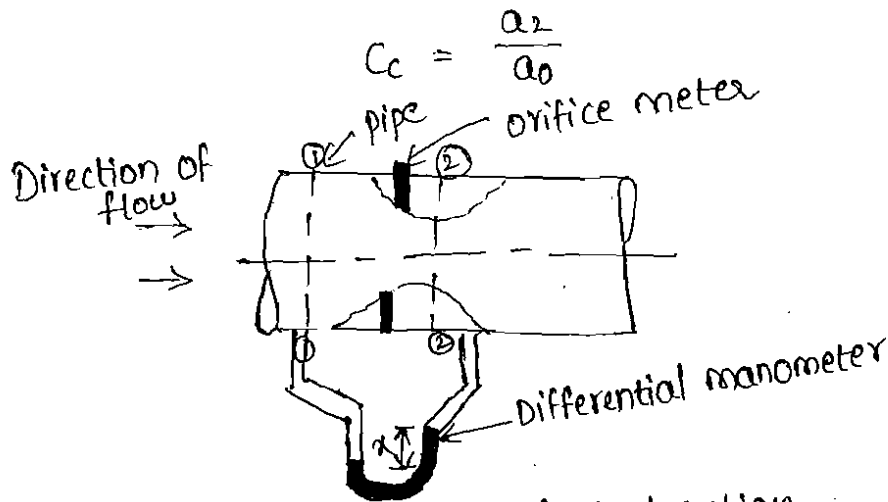
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$2gh = v_2^2 - v_1^2$$

$$v_2 = \sqrt{2gh + v_1^2} \quad \text{--- (1)}$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have



where C_c = co-efficient of contraction

$$a_2 = a_0 \times C_c$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2$$

$$v_1 = \frac{a_2}{a_1} v_2$$

$$v_1 = \frac{a_0 C_c}{a_1} v_2$$

Substitute v_1 in eq (1)

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

$$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2$$

$$v_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

⑤

The Discharge $Q = v_2 \times a_2 = v_2 \times a_0 c_c$

$$= \frac{a_0 c_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}} \quad \text{--- (2)}$$

The above equation is simplified by using

$$c_d = c_c \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}$$

$$c_c = c_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

Substitute c_c in eq (2).

$$Q = a_0 \times c_d \frac{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 c_c^2}}$$

$$= \frac{c_d a_0 \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}$$

$$= \frac{c_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

Where c_d = co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

problem

- ① An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when the co-efficient of discharge of orifice meter = 0.64.

Soln

Given

$$\text{Dia. of orifice, } d_o = 15 \text{ cm}$$

$$a_o = \frac{\pi}{4} (15)^2 = 176.7 \text{ cm}^2$$

$$\text{Dia. of pipe } d_i = 30 \text{ cm}$$

$$a_i = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$$

$$\text{Sp. gr. of oil } S_o = 0.9$$

$$x = 50 \text{ cm of mercury}$$

$$h = x \left[\frac{S_g}{S_o} - 1 \right] = 50 \left[\frac{13.6}{0.9} - 1 \right] \text{ cm of oil}$$

$$= 50 \times 14.11 = 705.5 \text{ cm of oil}$$

$$C_d = 0.64$$

Rate of flow, Q is

$$Q = C_d \frac{a_o a_i}{\sqrt{a_i^2 - a_o^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= 137414.25 \text{ cm}^3/\text{s}$$

$$Q = 137.414 \text{ lit/s}$$

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⑥

Pitot-tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in fig. The lower end, which is bent through 90° is directed in the upstream direction. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at same level

P_1 = intensity of pressure at point (1)

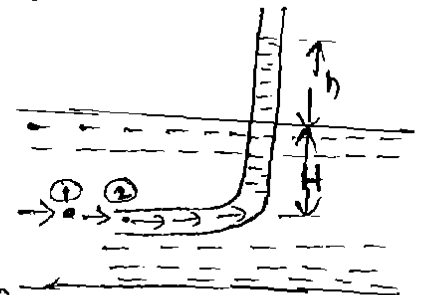
V_1 = velocity of flow at (1)

P_2 = pressure at point (2)

V_2 = velocity at point (2) which is zero

H = depth of tube in liquid

h = rise of liquid in tube above free surface.



Applying Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$ and $V_2 = 0$.

$$\frac{P_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{P_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values

$$H + \frac{v_1^2}{2g} = (h + H)$$

$$h = \frac{v_1^2}{2g}$$

$$v_1 = \sqrt{2gh}$$

Actual velocity

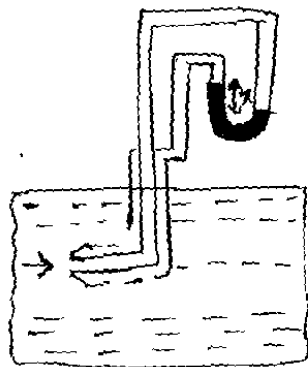
$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = co-efficient of pitot-tube

$$\therefore \text{velocity at any point } v = C_v \sqrt{2gh}$$

Pitot Static tube:

Pitot static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in fig. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of levels of the manometer liquid. Say α . Then $h = \alpha \left[\frac{\rho_f}{\rho} - 1 \right]$.



Problems:

1. A pitot static tube is used to measure the velocity of water in a pipe. The Stagnation pressure head is 6m and Static pressure head is 5m. Calculate the velocity of flow assuming the Co-efficient of tube equal to 0.98.

Sol:-

Given:

Stagnation pressure head, $h_s = 6\text{ m}$ Static pressure head, $h_t = 5\text{ m}$

$$h = 6 - 5 = 1\text{ m}$$

$$\text{Velocity of flow, } v = C_v \sqrt{2gh} = 0.98 \sqrt{2 \times 9.81 \times 1} = 4.34\text{ m/s.}$$

2. Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tapings of pitot-tube is 100 mm. Take Co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.

Sol:-

Given

$$x = 100\text{ mm} = 0.1\text{ m}$$

$$S_o = 0.8$$

$$S_g = 13.6$$

$$C_v = 0.98$$

$$h = x \left[\frac{S_g}{S_o} - 1 \right]$$

$$= 0.1 \left[\frac{13.6}{0.8} - 1 \right] = 1.6\text{ m of oil}$$

$$\therefore \text{Velocity of flow} = C_v \sqrt{2gh} = 0.98 \sqrt{2 \times 9.81 \times 1.6}$$

$$= 5.49\text{ m/s.}$$

Orifices:-

Definition:- orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side (or) at the bottom of a tank, through which a fluid is flowing. A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank (or) vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

Classification of orifices:-

The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upstream edge. The following are the important classifications:

- 1) The orifices are classified as small orifice (or) large orifice. Depending upon the size of orifice and head of liquid from the centre of the orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice it is known as large orifice.
- 2) The orifices are classified as i) circular orifice, ii) triangular orifice, iii) rectangular orifice and iv) square orifice depending upon their cross-sectional areas.
- 3) The orifices are classified as i) sharp-edged orifice and ii) bell-mouthed orifice depending upon the shape of

upstream edge of the orifices.

(8)

w) The orifices are classified as i) free discharging orifices and ii) drowned (or) sub-merged orifices depending upon the nature of discharge.

The sub-merged orifices are further classified as a) fully sub-merged orifices and ii) partially sub-merged orifices.

FLOW THROUGH AN ORIFICE:-

consider a tank filled with a circular orifice in one of its sides as shown in fig. Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section C-C, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called vena-contracta. Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

consider 2 points 1 and 2 as shown in fig. point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H . Applying Bernoulli's equation at points 1 and 2.

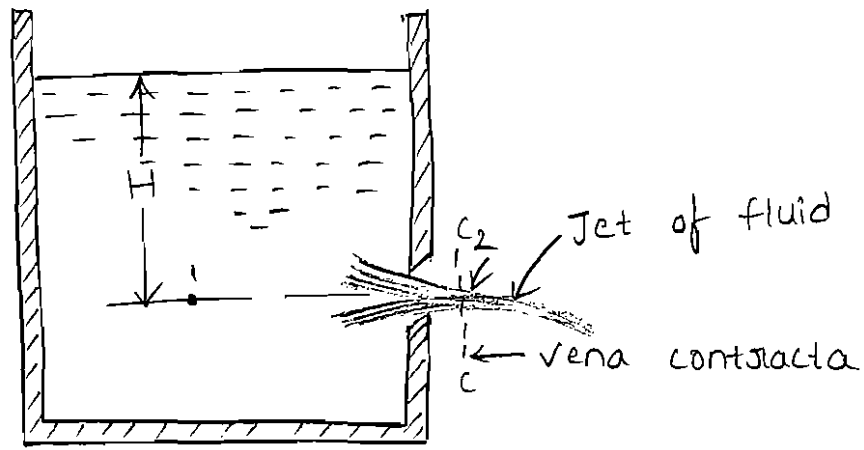


fig. Tank with an orifice

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad [W = \rho g]$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $\frac{P_1}{\rho g} = H$

$$\frac{P_2}{\rho g} = 0 \text{ (atmospheric pressure)}$$

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet (or) liquid.

$$H + 0 = 0 + \frac{v_2^2}{2g}$$

$$v_2^2 = 2gH$$

$$v_2 = \sqrt{2gH}$$

This is theoretical velocity. Actual velocity will be less than this value.

Hydraulic co-efficients:-

The hydraulic co-efficients are

- 1) co-efficient of velocity (C_v)
- 2) co-efficient of contraction (C_c)
- 3) co-efficient of discharge (C_d)

1) co-efficient of velocity (C_v):- It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by C_v and mathematically, (C_v) is given by

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{v}{\sqrt{2gH}}, \text{ where } v = \text{actual velocity}$$

$$\sqrt{2gH} = \text{Theoretical velocity}$$

The value of C_v varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place. Generally the value of $C_v = 0.98$ is taken for sharp-edged orifices.

2) co-efficient of contraction (C_c):- It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c .

Let $a =$ area of orifice and

$a_c =$ area of jet at vena-contracta

then
$$C_c = \frac{\text{Area of jet at vena-contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken as 0.64.

3) Co-efficient of discharge (C_d):- It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d . If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given by

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$C_d = C_v \times C_c$$

∴ The value of C_d varies from 0.61 to 0.65, for general purpose the value of C_d is taken as 0.62.

Problems:-

1) The head of water over an orifice of diameter 40 mm is 10m. find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$

A) Given

Head (H) = 10cm

Dia. of orifice (d) = 40mm = 0.04m

$$\text{Area (a)} = \frac{\pi}{4} (0.04)^2$$

$$= 0.001256 \text{ m}^2$$

$C_d = 0.6$

$C_v = 0.98$

i) $\frac{\text{Actual discharge}}{\text{Theoretical discharge}} = 0.6$

But theoretical discharge = $V_{th} \times \text{Area of orifice}$

V_{th} = Theoretical velocity,

Where $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10}$

$$= 14 \text{ m/s}$$

Theoretical discharge = 14×0.001256

$$= 0.01758 \text{ m}^3/\text{s}$$

Actual discharge = $0.6 \times \text{Theoretical discharge}$

$$= 0.6 \times 0.01758$$

$$= 0.01054 \text{ m}^3/\text{s}$$

ii) Actual velocity = ?

$$\frac{\text{Actual velocity}}{\text{Theoretical velocity}} = C_v = 0.98$$

$$\begin{aligned} \text{Actual velocity} &= 0.98 \times \text{Theoretical velocity} \\ &= 0.98 \times 14 \\ &= 13.72 \text{ m/s} \end{aligned}$$

EXPERIMENTAL DETERMINATION OF HYDRAULIC CO-EFFICIENTS

Determination of co-efficient of Discharge (C_d)

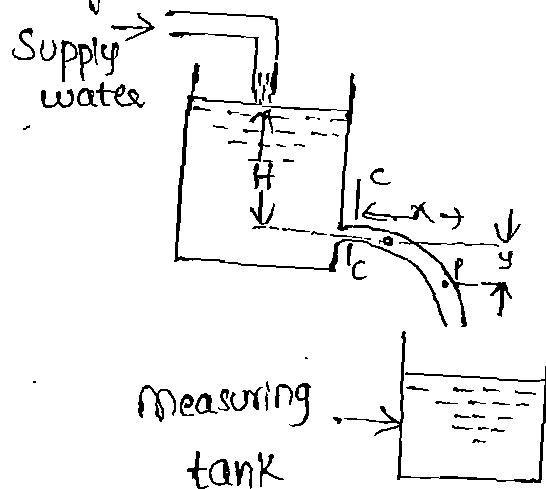
The water is allowed to flow through an orifice fitted to a tank under a constant head, H as shown in fig. The water is collected in a measuring tank for a known time, t . The height of water in the measuring tank is noted down. Then actual discharge through orifice,

$$Q = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time (t)}}$$

Time (t)

And theoretical discharge = area of orifice $\times \sqrt{2gH}$

$$C_d = \frac{Q}{a\sqrt{2gH}}$$



* Determination of Co-efficient of velocity (C_v):

Let C-c represents the vena-contracta of a jet of water coming out from an orifice under constant head H as shown in fig. Consider a liquid particle which is at vena-contracta at any time and takes the position at p along the jet in time 't'.

Let x = horizontal distance travelled by the particle in time 't'.

y = vertical distance between p and c-c

V = actual velocity of jet at vena-contracta.

Then horizontal distance, $x = Vxt$ — ①

and vertical distance, $y = \frac{1}{2}gt^2$ — ②

From ① $t = \frac{x}{V}$

Substitute t in ② we get

$$y = \frac{1}{2}g\left(\frac{x}{V}\right)^2$$

$$V^2 = \frac{gx^2}{2y}$$

$$V = \sqrt{\frac{gx^2}{2y}}$$

But theoretical velocity, $V_{th} = \sqrt{2gH}$

∴ Co-efficient of velocity, $C_v = \frac{V}{V_{th}} = \sqrt{\frac{gx^2}{2y}} \times \frac{1}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$

$$C_v = \frac{x}{\sqrt{4yH}}$$

* Determination of Co-efficient of contraction (C_c)

The Co-efficient of contraction is determined from the equation as

$$C_d = C_v \times C_c$$

$$C_c = \frac{C_d}{C_v}$$

problems

- ① A jet of water, issuing from a sharp-edged vertical orifice under a constant head of 10.0 cm at a certain point, has the horizontal and vertical co-ordinates measure from the vena-contracta as 20.0 cm and 10.5 cm respectively. find the value of C_v . Also find the value of C_c if $C_d = 0.60$

Given

Sol:-

Head, $H = 10.0 \text{ cm}$

Horizontal distance, $x = 20.0 \text{ cm}$

Vertical distance, $y = 10.5 \text{ cm}$

$C_d = 0.6$

The value of C_v is given by

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{20.0}{\sqrt{4 \times 10.5 \times 10.0}} = \frac{20}{20.493} = 0.976$$

The value of C_c is given by

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{0.976} = 0.6147 = 0.615$$

② Water discharge at the rate of 98.2 litres/s through a 120 mm diameter vertical sharp-edged orifice placed under constant head of 10 metres. A point, on the jet, measured from the vena-contracta of the jet has co-ordinates 4.5 metres horizontal and 0.54 metres vertical. Find the coefficient C_v , C_c and C_d of the orifice. (12)

Sol:-

Given:

$$\text{Discharge } Q = 98.2 \text{ lit/s} = 0.0982 \text{ m}^3/\text{s}$$

$$\text{Dia. of orifice, } d = 120 \text{ mm} = 0.12 \text{ m}$$

$$\therefore \text{Area of orifice, } a = \frac{\pi}{4} (0.12)^2 = 0.01131 \text{ m}^2$$

$$\text{Head, } H = 10 \text{ m}$$

$$\text{Horizontal distance } x = 4.5 \text{ m}$$

$$\text{Vertical distance } y = 0.54 \text{ m}$$

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$$

$$Q_{th} = V_{th} \times \text{area of orifice}$$

$$= 14 \times 0.01131$$

$$= 0.158 \text{ m}^3/\text{s}$$

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q}{Q_{th}} = \frac{0.0982}{0.1583} = 0.62$$

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = 0.968$$

$$C_c = \frac{C_d}{C_v} = \frac{0.62}{0.968} = 0.64$$

FLOW THROUGH LARGE ORIFICES

If the head of liquid is less than 5 times the depth of the orifice, the orifice is called large orifice. In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge can be calculated by $Q = C_d \times a \times \sqrt{2gh}$. But in case of a large orifice, the velocity is not constant over the entire cross-section of jet and hence Q cannot be calculated by $Q = C_d \times a \times \sqrt{2gh}$.

DISCHARGE THROUGH LARGE RECTANGULAR ORIFICE:

Consider a large rectangular orifice in one side of the tank discharging freely into atmosphere under a constant head H as shown in figure.

- let H_1 = height of liquid above top edge of orifice
 H_2 = height of liquid above bottom edge of orifice
 b = breadth of orifice
 d = depth of orifice = $H_2 - H_1$
 C_d = co-efficient of discharge.

Consider an elementary horizontal strip of depth ' dh ' at a depth of ' h ' below the free surface of the liquid in the tank.

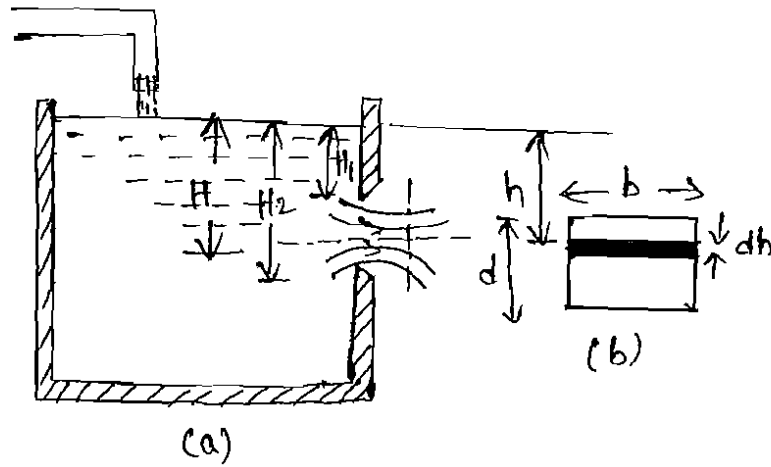


fig. Large rectangular orifice

$$\text{Area of strip} = b \times dh$$

$$\text{Theoretical velocity of water through strip} = \sqrt{2gh}$$

\therefore Discharge through elementary strip is given

$$dQ = C_d \times \text{Area of strip} \times \text{velocity}$$

$$= C_d \times b \times dh \times \sqrt{2gh}$$

$$= C_d b \sqrt{2gh} dh$$

By integrating the above equation between the limits H_1 and H_2 , the total discharge through the whole orifice is obtained.

$$Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh} dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h} dh$$

$$= C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right].$$

Problem

- ① A rectangular orifice, 1.5 m wide and 1.0 m deep is discharging water from a tank. If the water level in the tank is 3 m above the top edge of the orifice, find the discharge through the orifice. Take the coefficient of discharging for the orifice = 0.6.

Sol:

Given

Width of orifice, $b = 1.5 \text{ m}$

Depth of orifice $d = 1 \text{ m}$

$H_1 = 3 \text{ m}$

$H_2 = H_1 + d = 3 + 1 = 4 \text{ m}$

$C_d = 0.6$

Discharge Q is given by

$$\begin{aligned} Q &= \frac{2}{3} \times C_d \times b \times \sqrt{2g} [H_2^{3/2} - H_1^{3/2}] \\ &= \frac{2}{3} \times 0.6 \times 1.5 \times \sqrt{2 \times 9.81} [4.0^{1.5} - 3.0^{1.5}] \text{ m}^3/\text{s} \\ &= 2.657 [8.0 - 5.196] \text{ m}^3/\text{s} \\ &= 7.45 \text{ m}^3/\text{s} \end{aligned}$$

Mouthpiece :

A mouthpiece is a short length of a pipe which is two or three times its diameter in length.

Classification of Mouthpieces :

1. The mouthpieces are classified as (i) External mouthpiece or (ii) Internal mouthpiece depending upon their position with respect to the tank or vessel to which they are fitted.
2. The mouthpiece are classified as (i) cylindrical mouthpiece (ii) Convergent mouthpiece (iii) Convergent - divergent mouthpiece depending upon their shapes.
3. The mouthpieces are classified as (i) mouthpieces running full (ii) Mouthpieces running free. depending upon the nature of discharge at the outlet of the mouthpiece. This classification is only for internal mouthpieces which are known Borda's or Re-entrant mouthpieces. A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as running full.

FLOW THROUGH AN EXTERNAL CYLINDRICAL MOUTHPIECE

Consider a tank having an external cylindrical mouthpiece of cross-sectional area a_1 , attached to one of its sides as shown in fig. The jet of liquid entering the mouthpiece contracts to form a vena-contracta at a section C-C. Beyond this section, the jet again expands and fills the mouthpiece completely.

Let H = Height of liquid above the centre of mouthpiece.

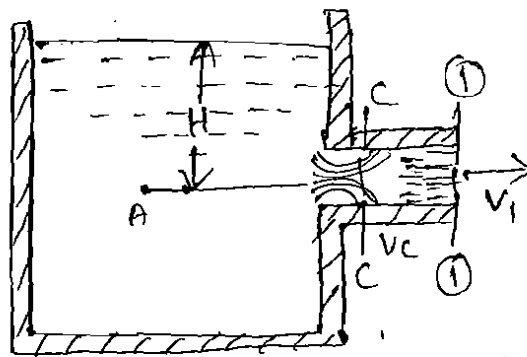
v_c = velocity of liquid at C-C section

a_c = Area of flow at vena-contracta

v_1 = velocity of liquid at outlet

a_1 = area of mouthpiece at outlet

C_c = Co-efficient of contraction.



External cylindrical mouthpiece

Applying continuity equation at C-C and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$v_c = \frac{a_1 v_1}{a_c}$$

$$= \frac{v_1}{(a_c/a_1)}$$

But $\frac{a_c}{a_1} = C_c =$ co-efficient of contraction

Taking $C_c = 0.62$, we get $\frac{a_c}{a_1} = 0.62$

$$V_c = \frac{V_1}{0.62}$$

The jet of liquid from section c-c suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L^* which is given as

$$h_L = \frac{(V_c - V_1)^2}{2g}$$

$$\begin{aligned} \text{But } V_c &= \frac{V_1}{0.62} \text{ hence } h_L = \frac{\left(\frac{V_1}{0.62} - V_1\right)^2}{2g} \\ &= \frac{V_1^2}{2g} \left[\frac{1}{0.62} - 1\right]^2 \\ &= \frac{0.375 V_1^2}{2g} \end{aligned}$$

Applying Bernoulli's equation to point A and (1)-(1)

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, V_A is negligible,

$$\frac{P_1}{\rho g} = \text{atmospheric pressure} = 0$$

$$\therefore H + 0 = 0 + \frac{V_1^2}{2g} + 0.375 \frac{V_1^2}{2g}$$

$$H = 1.375 \frac{v_1^2}{2g}$$

$$v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855 \sqrt{2gH}$$

Theoretical velocity of liquid at outlet is

$$v_{th} = \sqrt{2gH}$$

∴ Co-efficient of velocity for mouthpiece:

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855 \sqrt{2gH}}{\sqrt{2gH}} = 0.855$$

C_c for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus

$$C_d = C_c \times C_v = 1.0 \times 0.855 = 0.855$$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice and so discharge through mouthpiece will be more.

Problem:

- ① Find the discharge from a 100 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 4 metres.

Given

$$\text{Dia. of mouthpiece} = 100 \text{ mm} = 0.1 \text{ m}$$

$$\therefore \text{Area, } a = \frac{\pi}{4} (0.1)^2 = 0.007854 \text{ m}^2$$

Sol:-

Head, $H = 4\text{ m}$

C_d for mouthpiece $= 0.855$

$$\begin{aligned}\therefore \text{Discharge} &= C_d \times \text{Area} \times \text{velocity} \\ &= 0.855 \times a \times \sqrt{2gH} \\ &= 0.855 \times 0.007854 \times \sqrt{2 \times 9.81 \times 4} \\ &= 0.5948 \text{ m}^3/\text{s}\end{aligned}$$

② An external cylindrical mouthpiece of diameter 150 mm is discharging water under a constant head of 6 m. Determine the discharge and absolute pressure head of water at vena-contracta. Take $C_d = 0.855$ and C_c for vena-contracta $= 0.62$. Atmospheric pressure head $= 10.3$ m of water.

Given

Dia. of mouthpiece $d = 150 \text{ mm} = 0.15 \text{ m}$

Area $a = \frac{\pi}{4} (0.15)^2 = 0.01767 \text{ m}^2$

Head, $H = 6 \text{ m}$

$C_d = 0.855$

C_c at vena-contracta $= 0.62$

Atmospheric pressure head $H_a = 10.3 \text{ m}$

$$\begin{aligned}\therefore \text{Discharge} &= C_d \times a \times \sqrt{2gH} \\ &= 0.855 \times 0.01767 \times \sqrt{2 \times 9.81 \times 6} \\ &= 0.1639 \text{ m}^3/\text{s}\end{aligned}$$

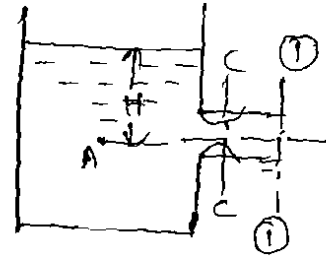
Pressure Head at Vena-Contracta

Applying Bernoulli's equation at A and C-C, we get

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} + Z_C$$

But $\frac{P_A}{\rho g} = H_a + H, \quad V_A = 0$

$$Z_A = Z_C$$



$$H_a + H + 0 = \frac{P_C}{\rho g} + \frac{V_C^2}{2g} = H_c + \frac{V_C^2}{2g}$$

$$H_c = H_a + H - \frac{V_C^2}{2g}$$

But $V_C = \frac{V_1}{0.62}$

$$H_c = H_a + H - \left(\frac{V_1}{0.62}\right)^2 \times \frac{1}{2g}$$

$$= H_a + H - \frac{V_1^2}{2g} \times \frac{1}{(0.62)^2}$$

$$H = 1.375 \frac{V_1^2}{2g}$$

$$\frac{V_1^2}{2g} = \frac{H}{1.375} = 0.7272 H$$

$$H_c = H_a + H - 0.7272 H \times \frac{1}{(0.62)^2}$$

$$= H_a + H - 1.89 H$$

$$= H_a - 0.89 H$$

$$= 10.3 - 0.89 \times 6$$

$$= 10.3 - 5.34$$

$$= 4.96 \text{ m}$$

[$H_a = 10.3$
and
 $H = 6.0$]

FLOW THROUGH INTERNAL OR RE-ENTRANT OR BORDA'S MOUTHPIECE

A short cylindrical tube attached to an orifice in such a way that the tube projects inwardly to a tank is called an internal mouthpiece. It is also called re-entrant or Borda's mouthpiece. If the length of the tube is equal to its diameter, the jet of liquid comes out from mouthpiece without touching the sides of tube as shown in fig. The mouthpiece is known as running free. But if the length of the tube is about 3 times its diameter, the jet comes out with its diameter equal to the diameter of mouthpiece at outlet as shown in fig. The mouthpiece is said to be running full.

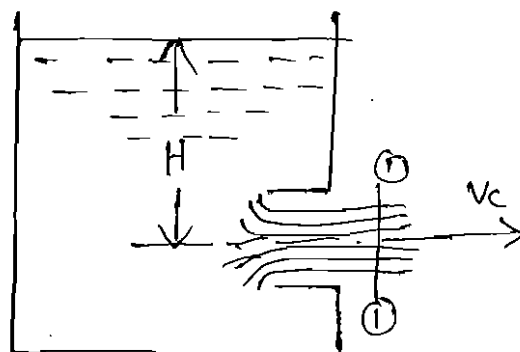
(i) Borda's Mouthpiece Running Free :

Let H = height of liquid above the mouthpiece

a = area of mouthpiece

a_c = area of contracted jet in the mouthpiece

v_c = velocity through mouthpiece.



Running free

The flow of fluid through mouthpiece is taking place due to the pressure force exerted by the fluid on the entrance section of the mouthpiece. As the area of the mouthpiece is 'a' hence total pressure force on entrance

$$= \rho g \cdot a \cdot h$$

where $h =$ distance of C.G of area 'a' from free surface $= H$

$$= \rho g \cdot a \cdot H \quad \text{--- (1)}$$

According to Newton's second law of motion, the net force is equal to the rate of change of momentum.

Now mass of liquid flowing /sec $= \rho \times a_c \times v_c$

The liquid is initially at rest and hence initial velocity is zero and final velocity of fluid is v_c .

\therefore Rate of change of momentum $=$ mass of liquid flowing /sec \times [final velocity - initial velocity]

$$= \rho a_c \times v_c [v_c - 0]$$

$$= \rho a_c v_c^2 \quad \text{--- (2)}$$

Equating (1) and (2) we get

$$\rho g a H = \rho a_c v_c^2 \quad \text{--- (3)}$$

Applying Bernoulli's equation

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

We have

$$z = H, \quad z_1 = 0, \quad \frac{P}{\rho g} = \frac{P_1}{\rho g} = P_{atmosp.} = 0$$

$$V_1 = V_c, \quad V = 0$$

$$0 + 0 + H = 0 + \frac{V_c^2}{2g} + 0$$

$$H = \frac{V_c^2}{2g}$$

$$V_c = \sqrt{2gH}$$

Substituting in (3)

$$\rho g \cdot a \cdot H = \rho \cdot a_c \cdot 2g \cdot H$$

$$a = 2a_c$$

$$\frac{a_c}{a} = \frac{1}{2} = 0.5$$

$$\therefore \text{Co-efficient of contraction } C_c = \frac{a_c}{a} = 0.5$$

Since there is no loss of head, Co-efficient of velocity $C_v = 1.0$

$$\therefore C_d = C_c \times C_v = 0.5 \times 1.0 = 0.5$$

$$\text{Discharge } Q = C_d a \sqrt{2gH} = 0.5 a \sqrt{2gH}$$

(ii) Borda's Mouthpiece running full:

Let H = height of liquid above the mouthpiece

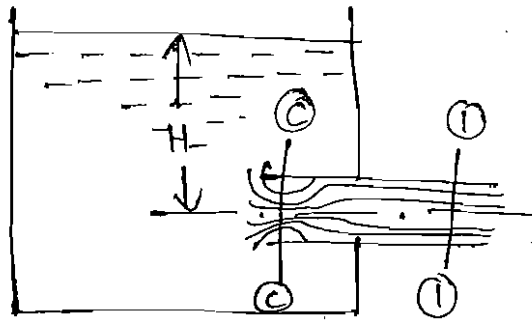
V_1 = velocity at outlet or at (1)-(1) of mouthpiece

a = area of mouthpiece

a_c = area of flow at C-C

V_c = velocity of liquid at Vena-contracta or at

C-C



Running full

Thus there will be loss of head

$$h_L = \frac{(v_c - v_1)^2}{2g} \quad \text{--- (1)}$$

From continuity equation

$$a_c v_c = a_1 v_1$$

$$v_c = \frac{a_1}{a_c} v_1 = \frac{v_1}{a_c/a_1} = \frac{v_1}{c_c} = \frac{v_1}{0.5}$$

$$\boxed{v_c = 2v_1}$$

Substituting v_c in (1)

$$h_L = \frac{(2v_1 - v_1)^2}{2g} = \frac{v_1^2}{2g}$$

Applying Bernoulli's equation

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

$$0 + 0 + H = 0 + \frac{v_1^2}{2g} + 0 + \frac{v_1^2}{2g}$$

$$H = \frac{v_1^2}{2g} + \frac{v_1^2}{2g}$$

$$H = \frac{v_1^2}{g}$$

$$V_1 = \sqrt{gH}$$

(19)

But theoretical velocity $V_{th} = \sqrt{2gH}$

$$\therefore \text{Co-efficient of velocity, } C_v = \frac{V_1}{V_{th}} = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}} = 0.707$$

As the area of the jet at outlet is equal to the area of the mouthpiece, hence Co-efficient of Contraction = 1

$$C_d = C_c \times C_v = 1.0 \times 0.707 = 0.707$$

$$\therefore \text{Discharge } Q = C_d a \sqrt{2gH} = 0.707 a \sqrt{2gH}$$

Problem:

1. An internal mouthpiece of 80 mm diameter is discharging water under a constant head of 8 metres. Find the discharge through mouthpiece, when
- (i) Mouthpiece is running free (ii) Mouthpiece is running full.

Sol:-

Given:

Dia. of mouthpiece, $d = 80 \text{ mm} = 0.08 \text{ m}$

$$\therefore \text{Area } a = \frac{\pi}{4} (0.08)^2 = 0.005026 \text{ m}^2$$

Constant head, $H = 4 \text{ m}$

(i) Mouthpiece running free

The discharge Q is given by

$$Q = 0.5 \times a \times \sqrt{2gH}$$

$$= 0.5 \times 0.005026 \times \sqrt{2 \times 9.81 \times 4}$$

$$= 0.02226 \text{ m}^3/\text{s}$$

$$= 22.26 \text{ lit/s}$$

(ii) mouthpiece running full:

The discharge Q is given by

$$Q = 0.707 \times a \times \sqrt{2gH}$$

$$= 0.707 \times 0.005026 \times \sqrt{2 \times 9.81 \times 4}$$

$$= 0.03147 \text{ m}^3/\text{s}$$

$$= 31.47 \text{ lit/s}$$

* Notch:

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

* Weir :

(20)

A weir is a concrete or masonry structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open-channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonry structure.

* Nappe or vein :

The sheet of water flowing through a notch or over a weir is called Nappe or vein.

* Crest or Sill :

The bottom edge of a notch or a top of a weir over which the water flows is known as the sill or crest.

→ CLASSIFICATION OF NOTCHES AND WEIRS :

The notches are classified as;

1. According to the shape of the opening :

(a) Rectangular notch

(b) Triangular notch

(c) Trapezoidal notch and

(d) Stepped notch.

2. According to the effect of the Sides on the nappe:

(a) Notch with end contraction.

(b) Notch without end contraction or Suppressed notch

Weirs are classified according to the Shape of the opening, the shape of the Crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(a) According to the Shape of the opening:

(i) Rectangular weir (ii) Triangular weir

(iii) Trapezoidal weir (Cipolletti weir)

(b) According to the Shape of the Crest:

(i) Sharp-crested weir

(ii) Broad-crested weir

(iii) Narrow-crested weir

(iv) ogee-shaped weir.

(c) According to the effect of Sides on the emerging

nappe:

(i) Weir with end contraction

(ii) Weir without end contraction.

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

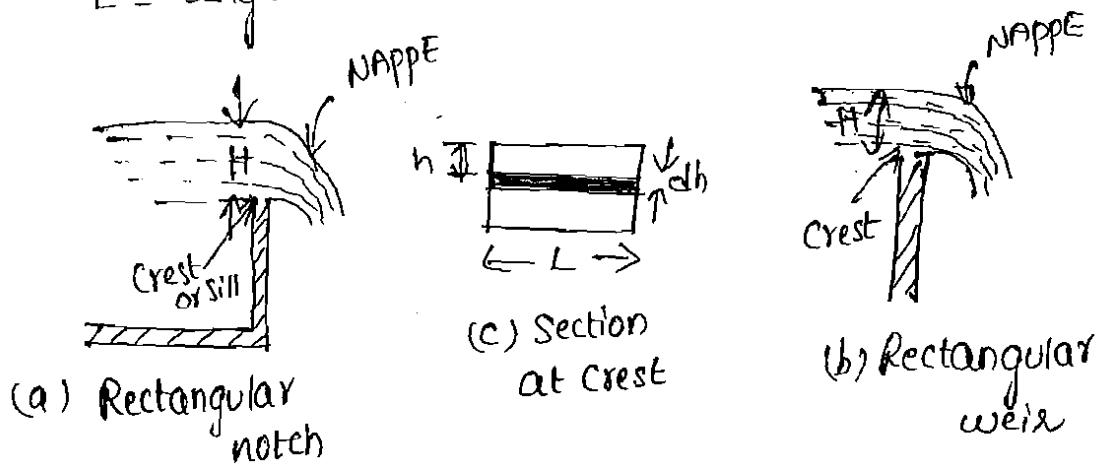
(21)

The expression for discharge over a rectangular notch or weir is the same.

Consider a rectangular notch or weir provided in a channel carrying water as shown in fig

Let H = head of water over the crest

L = length of the notch or weir



For finding the discharge of water flowing over the weir or notch. Consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in fig (c).

The area of Strip = $L \times dh$

and theoretical velocity of water flowing through

$$\text{Strip} = \sqrt{2gh}$$

The discharge dQ , through strip is

$$dQ = C_d \times \text{Area of Strip} \times \text{Theoretical Velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh}$$

where C_d = Co-efficient of discharge.

The total discharge Q , for the whole notch or weir is determined by integrating equation and limits 0 and H .

$$Q = \int_0^H C_d \cdot L \cdot \sqrt{2gh} \, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} \, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{1/2+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2}$$

problems

- ① Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.

Given:

Sol:-

Length of the notch, $L = 2 \text{ m}$

Head over notch $H = 300 \text{ mm} = 0.3 \text{ m}$

$C_d = 0.6$

$$\text{Discharge } Q = \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2}$$

$$\begin{aligned}
 &= \frac{2}{3} \times 0.6 \times 2 \times \sqrt{2 \times 9.81} \times [0.3]^{1.5} \text{ m}^3/\text{s} \\
 &= 3.5435 \times 0.1643 \\
 &= 0.582 \text{ m}^3/\text{s}.
 \end{aligned}$$

② Determine the height of a rectangular weir of length 6m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8m and discharge is 2000 litres/s. Take $C_d = 0.6$ and neglect end contractions.

Sol:-

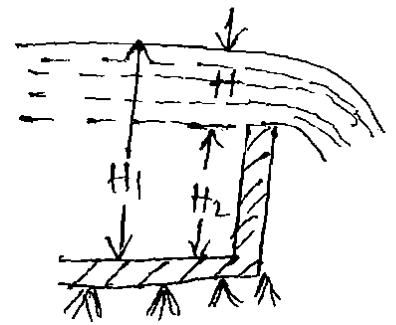
Given:

length of weir, $L = 6 \text{ m}$

Depth of water, $H_1 = 1.8 \text{ m}$

Discharge, $Q = 2000 \text{ lit/s} = 2 \text{ m}^3/\text{s}$

$C_d = 0.6$



let H is height of water above the crest of weir

$H_2 =$ height of weir.

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

$$2 = \frac{2}{3} \times 0.6 \times 6 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623 H^{3/2}$$

$$H^{3/2} = \frac{2}{10.623}$$

$$H = \left(\frac{2}{10.623} \right)^{2/3} = 0.328 \text{ m}$$

$$\begin{aligned}
 \therefore \text{Height of weir, } H_2 &= H_1 - H \\
 &= \text{Depth of water on upstream side} - H \\
 &= 1.8 - 0.328 \\
 &= 1.472 \text{ m}
 \end{aligned}$$

→ DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

Let H = head of water above the V-notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of 'h' from the free surface of water as shown in fig.

we have

$$\tan \frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h) \tan \frac{\theta}{2}$$

$$\text{Width of strip} = AB = 2AC = 2(H-h) \tan \frac{\theta}{2}$$

$$\therefore \text{Area of strip} = 2(H-h) \tan \frac{\theta}{2} \times dh$$

$$\text{The theoretical velocity of water through strip} = \sqrt{2gh}$$

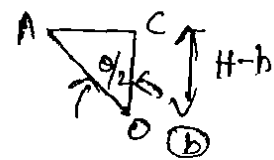
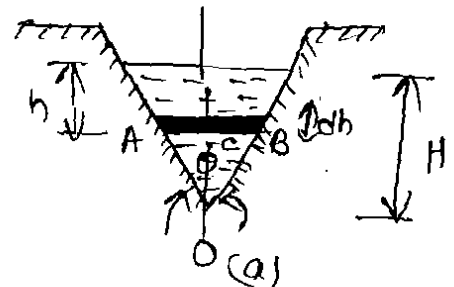
\therefore Discharge, through the strip

$$dQ = C_d \times \text{Area of strip} \times \text{velocity}$$

$$= C_d \times 2(H-h) \tan \frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d (H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$\therefore \text{Total discharge } Q = \int_0^H 2C_d (H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh$$



$$\begin{aligned}
&= 2 C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (H-h) h^{1/2} dh \\
&= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (Hh^{1/2} - h^{3/2}) dh \\
&= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H \\
&= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H \cdot H^{3/2} - \frac{2}{5} H^{5/2} \right] \\
&= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right]
\end{aligned}$$

$$Q = \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90^\circ, \therefore \tan \frac{\theta}{2} = 1$$

Discharge $Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$

$$Q = 1.417 H^{5/2}$$

Problem

- ① Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m . Assume $C_d = 0.6$.

Sol:-

Given

Angle of V-notch, $\theta = 60^\circ$

Head over notch, $H = 0.3 \text{ m}$

$C_d = 0.6$

$$\begin{aligned}
 Q &= \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \\
 &= \frac{8}{15} \times 0.6 \tan \frac{60^\circ}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2} \\
 &= 0.8182 \times 0.0493 \\
 &= 0.040 \text{ m}^3/\text{s}.
 \end{aligned}$$

ADVANTAGES OF TRIANGULAR NOTCH OR WEIR OVER RECTANGULAR NOTCH OR WEIR:

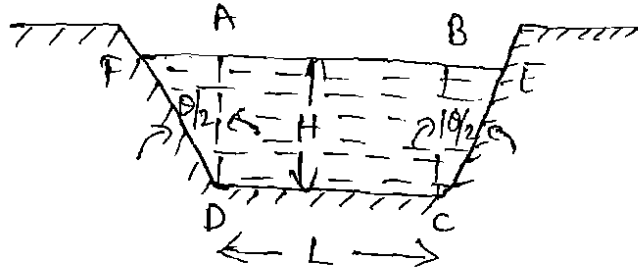
A triangular notch or weir is preferred to a rectangular weir or notch due to following reasons:

1. The expression for discharge for a right-angled v-notch or weir is very simple.
2. For measuring low discharge, a triangular notch gives more accurate results than a rectangular notch.
3. In case of triangular notch, only one reading i.e., H is required for the computation of discharge.
4. Ventilation of a triangular notch is not necessary.

→ DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR:

A trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

Let H = height of water over the notch
 L = length of the crest of the notch.



Trapezoidal notch

C_{d1} = Co-efficient of discharge for rectangular portion ABCD
 C_{d2} = Co-efficient of discharge for triangular portion [FAD and BCE]

The discharge through rectangular portion ABCD is given by

$$Q_1 = \frac{2}{3} \times C_{d1} \times L \times \sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches FAD and BCE is equal to the discharge through a single triangular notch of angle θ and it is given by equation as

$$Q_2 = \frac{8}{15} C_{d2} \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

\therefore Discharge through trapezoidal notch or weir FDCEF
 $= Q_1 + Q_2$

$$= \frac{2}{3} C_{d1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d2} \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

problem

1. Find the discharge through a trapezoidal notch which is 1m wide at top and 0.4 m at bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.

Soln

Given:

Top width, $AE = 1\text{ m}$

Base width, $CD = L = 0.4\text{ m}$

Head of water, $H = 0.2\text{ m}$

For rectangular portion, $C_{d1} = 0.62$

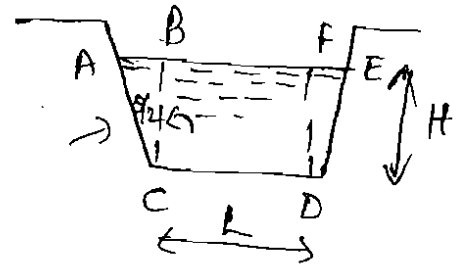
For triangular portion $C_{d2} = 0.60$

From $\triangle ABC$, we have

$$\begin{aligned}\tan\frac{\theta}{2} &= \frac{AB}{BC} = \frac{(AE - CD)/2}{H} \\ &= \frac{(1 - 0.4)/2}{0.3} \\ &= \frac{0.6/2}{0.3} \\ &= 1\end{aligned}$$

Discharge through trapezoidal notch is given by

$$\begin{aligned}Q &= \frac{2}{3} C_{d1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \\ &= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} \\ &\quad + \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}\end{aligned}$$



$$\begin{aligned}
 &= 0.06549 + 0.02535 \\
 &= 0.09084 \text{ m}^3/\text{s} \\
 &= 90.84 \text{ lit/s}
 \end{aligned}$$

→ DISCHARGE OVER A STEPPED NOTCH :

A stepped notch is a combination of rectangular notches.

The discharge through stepped notch is equal to the sum of the discharges through the different rectangular notches.

Consider a stepped notch as shown in fig.

Let H_1 = Height of water above the crest of notch 1

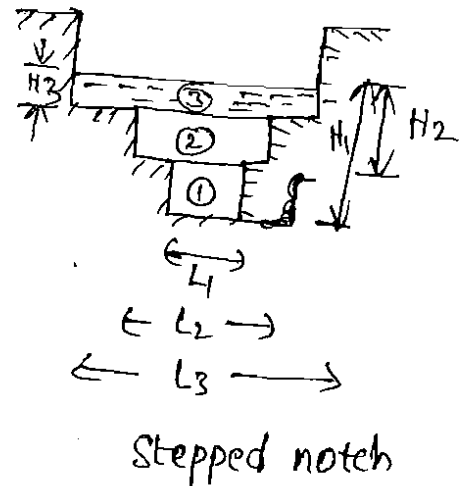
L_1 = length of notch 1

H_2, L_2 and H_3, L_3 are corresponding values for notches 2 and 3 respectively.

C_d = Co-efficient of discharge for all notches

∴ Total discharge $Q = Q_1 + Q_2 + Q_3$

$$\begin{aligned}
 \text{or } Q &= \frac{2}{3} C_d L_1 \sqrt{2g} [H_1^{3/2} - H_2^{3/2}] \\
 &+ \frac{2}{3} C_d L_2 \sqrt{2g} [H_2^{3/2} - H_3^{3/2}] \\
 &+ \frac{2}{3} C_d L_3 \sqrt{2g} \times H_3^{3/2}
 \end{aligned}$$



problem

- ① Fig shows a stepped notch. Find the discharge through the notch if C_d for all section = 0.62.

Sol:-

Given:

$$L_1 = 40 \text{ cm}, L_2 = 80 \text{ cm}$$

$$L_3 = 120 \text{ cm}$$

$$H_1 = 50 + 30 + 15 = 95 \text{ cm}$$

$$H_2 = 80 \text{ cm} \neq H_3 = 50 \text{ cm}$$

$$C_d = 0.62$$

Total discharge, $Q = Q_1 + Q_2 + Q_3$

$$\text{where } Q_1 = \frac{2}{3} C_d L_1 \sqrt{2g} [H_1^{3/2} - H_2^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 9.81} \times [95^{3/2} - 80^{3/2}]$$

$$= 732.26 [925.94 - 715.54]$$

$$= 154067 \text{ cm}^3/\text{s}$$

$$= 154.067 \text{ lit/s}$$

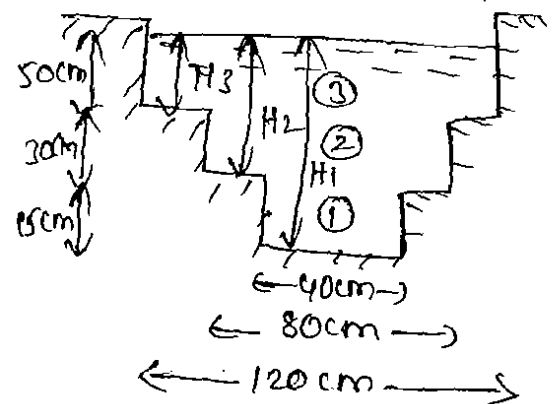
$$Q_2 = \frac{2}{3} C_d L_2 \sqrt{2g} [H_2^{3/2} - H_3^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 9.81} [80^{3/2} - 50^{3/2}]$$

$$= 1464.52 [715.54 - 353.55] \text{ cm}^3/\text{s}$$

$$= 530144 \text{ cm}^3/\text{s}$$

$$= 530.144 \text{ lit/s}$$



$$\begin{aligned}
 Q_3 &= \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2} \\
 &= \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 9.81} \times 50^{3/2} \\
 &= 776771 \text{ cm}^3/\text{s} \\
 &= 776.771 \text{ lit/s}
 \end{aligned}$$

$$\begin{aligned}
 Q &= Q_1 + Q_2 + Q_3 = 154.067 + 530.144 + 776.771 \\
 &= 1460.98 \text{ lit/s}
 \end{aligned}$$

VELOCITY OF APPROACH :

Velocity of approach is defined as the velocity with which the water approaches or reaches the weir or notch before it flows over it.

Thus if v_a is the velocity of approach, then an additional head h_a equal to $\frac{v_a^2}{2g}$ due to velocity of approach, is acting on the water flowing over the notch. Then initial height of water over the notch becomes $(H + h_a)$ and final height becomes equal to h_a . Then all the formulae are changed taking into consideration of velocity of approach.

$$V_a = \frac{Q}{\text{Area of channel}}$$

This velocity of approach is used to find an additional

$$\text{head } (h_a = \frac{v_a^2}{2g})$$

Discharge over a rectangular weir, with velocity of approach

$$= \frac{2}{3} \times C_d \times L \times \sqrt{2g} \left[(H + h_a)^{3/2} - h_a^{3/2} \right]$$

problem

- ① Find the discharge over a rectangular weir of length 100m. The head of water over the weir is 1.5m. The velocity of approach is given as 0.5 m/s. Take $C_d = 0.60$.

soln
=

Given

length of weir, $L = 100 \text{ m}$

Head of water, $H_1 = 1.5 \text{ m}$

Velocity of approach, $V_a = 0.5 \text{ m/s}$

$C_d = 0.6$

$$\therefore \text{Additional head, } h_a = \frac{V_a^2}{2g} = \frac{0.5 \times 0.5}{2 \times 9.81} = 0.0127 \text{ m}$$

The discharge, Q over a rectangular weir due to velocity of approach is given by

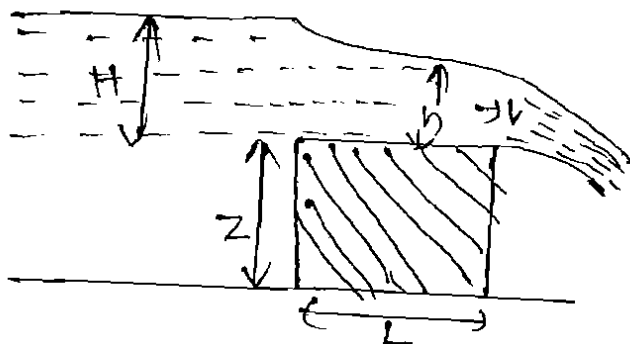
$$\begin{aligned} Q &= \frac{2}{3} \times C_d \times L \times \sqrt{2g} \left[(H_1 + h_a)^{3/2} - h_a^{3/2} \right] \\ &= \frac{2}{3} \times 0.6 \times 100 \times \sqrt{2 \times 9.81} \left[(1.5 + 0.0127)^{3/2} - 0.0127^{3/2} \right] \\ &= 177.16 \left[1.5127^{3/2} - 0.0127^{3/2} \right] \\ &= 177.16 \left[1.8605 - 0.00143 \right] \\ &= 329.35 \text{ m}^3/\text{s} \end{aligned}$$

→ DISCHARGE OVER A BROAD-CRESTED WEIR (197)

A weir having a wide crest is known as broad-crested weir.

Let H = height of water above the crest

L = length of the crest.



If $2L > H$, the weir is called broad-crested weir.

If $2L < H$, the weir is called a narrow-crested weir.

fig shows a broad-crested weir.

Let h = head of water at middle of weir which is constant.

v = velocity of flow over the weir.

Applying Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir

$$0 + 0 + H = 0 + \frac{v^2}{2g} + h$$

$$\frac{v^2}{2g} = H - h$$

$$v = \sqrt{2g(H - h)}$$

∴ The discharge over weir $Q = C_d \times \text{Area of flow} \times \text{velocity}$

$$= C_d \times L \times h \times \sqrt{2g(H-h)}$$

$$= C_d \times L \times \sqrt{2g(Hh^2 - h^3)} \quad \text{--- (1)}$$

The discharge will be maximum, if $(Hh^2 - h^3)$ is maximum

$$\frac{d}{dh} (Hh^2 - h^3) = 0$$

$$2hH - 3h^2 = 0$$

$$2H = 3h$$

$$h = \frac{2}{3} H$$

Substitute in eq (1)

$$Q_{\max} = C_d \times L \times \sqrt{2g \left[H \left(\frac{2}{3} H \right)^2 - \left(\frac{2}{3} H \right)^3 \right]}$$

$$= C_d \times L \times \sqrt{2g} \sqrt{H \times \frac{4}{9} H^2 - \frac{8}{27} H^3}$$

$$= C_d \times L \times \sqrt{2g} \sqrt{\frac{4}{9} H^3 - \frac{8}{27} H^3}$$

$$= C_d \times L \times \sqrt{2g} \sqrt{\frac{4}{27} H^3}$$

$$= C_d \times L \times \sqrt{2g} \times 0.3849 \times H^{3/2}$$

$$= 0.3849 \times \sqrt{2 \times 9.81} \times C_d \times L \times H^{3/2}$$

$$= 1.7047 \times C_d \times L \times H^{3/2}$$

$$Q_{\max} = 1.705 \times C_d \times L \times H^{3/2}$$

→ DISCHARGE OVER A NARROW-CRESTED WEIR

(24)

For a narrow-crested weir, $2L < H$. It is similar to a rectangular weir or notch hence Q is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

problem:

- ① (a) A broad-crested weir of 50 m length, has 50 cm height of water above its crest. Find the maximum discharge. Take $C_d = 0.60$. Neglect velocity of approach. (b) If the velocity of approach is to be taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 50 m^2 on the upstream side.

Sol:-

Given:

Length of weir, $L = 50 \text{ m}$

Head of water, $H = 50 \text{ cm} = 0.5 \text{ m}$

$C_d = 0.60$

(i) Neglecting velocity of approach.

Maximum discharge is given by

$$\begin{aligned} Q_{\text{max}} &= 1.705 \times C_d \times L \times H^{3/2} \\ &= 1.705 \times 0.6 \times 50 \times (0.5)^{3/2} \\ &= 18.084 \text{ m}^3/\text{s} \end{aligned}$$

Maximum discharge Q_{max} is given by

$$Q_{max} = 1.705 \times C_d \times L \times \left[(H + h_a)^{3/2} - h_a^{3/2} \right]$$
$$= 1.705 \times 0.6 \times 50 \times \left[(0.5 + 0.0066)^{1.5} - (0.0066)^{1.5} \right]$$

$$= 51.15 \left[0.3605 - 0.000536 \right]$$

$$= 18.412 \text{ m}^3/\text{s}$$